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FINITE-ACTION SOLUTIONS OF HIGHER-ORDER YANG-MILLS-HIGGS THEORY IN FOUR DIMENSIONS

2

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Abstract:

We study (generalized) Yang-Mills-Higgs theories with higher-order terms. We present topologically nontrivial finite-action solutions in a mini-model and discuss a more relevant model later. Although the ansatz we choose is not SO(4) symmetric it leads to SO(4) invariant action densities and is compatible with the equations of motion for a wide class of models.

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Introduction

Recently, there has been considerable activity in the study of gauge field theories in more than four dimensions. The more geometrically motivated approach[1)] is based on an action density that is at least quadratic in the curvature 4-form $F\wedge F$, and enjoys two advantageous properties: First, such theories are expected to have improved ultra-violet behaviour, and second, they circumvent the theorem[2)] that there are no finite-action Yang-Mills (YM) solutions in more than four dimensions. Indeed, recently such finite-action solutions were discovered[3)4)], which satisfy the self-duality equation of the curvature 4-form in eight dimensions, and it was further shown that similar solutions exist in all 4p dimensions[4)]. Subsequently this system was also studied from a group representational viewpoint[5)]. The other approach[6)] retains the YM dynamics, and is based on a linear constraint on the curvature 2-form which in eight dimensions (only) has an interesting expression in terms of octonionic structure constants. Henceforth in this paper, we shall be concerned only with the former type of theory[2)], and in particular with the system given in Ref. 4.

However, if we believe in more than four dimensions at all we must assume that at some point of the evolution of the universe the extra dimensions are spontaneously compactified. This makes it natural to study possible dimensionally reduced models. These models have Higgs fields in addition to gauge fields and in many cases nontrivial topology. Because of the higher-order terms, which can compensate a loss in the action from the rescaling of the lower-order terms, one would also expect to find finite-action (generalized) Yang-Mills-Higgs (YMH) solutions corresponding to this nontrivial topology. In this paper, we pursue this idea by examining a mini-model for pedagogical reasons first and then a more relevant model based on the dimensional reduction of a 8-dimensional theory[7]. The latter model is especially interesting because it is endowed by a nontrivial surface integral.

2. A mini-model

In this section, we analyze the model given by the Lagrangian

$$\begin{aligned}\mathcal{L} = &-\mathrm{tr}\, F_{\mu\nu}F_{\mu\nu} + \langle D_\mu\varphi, D_\mu\varphi\rangle \\ &+ (\langle\varphi,\varphi\rangle - 1)^2 + \mathrm{tr}\, F_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\end{aligned} \tag{2.1}$$

in four dimensions ($\mu, \nu, \ldots = 1,2,3,4$) which is just YMH theory plus the square of the 4-form curvature. We examine this model because it shares most of the technical features of the models introduced in Ref. 7. Our notation is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu], \tag{2.2a}$$

$$D_\mu\varphi = \partial_\mu\varphi - A_\mu\varphi, \tag{2.2b}$$

$$F_{\mu\nu\rho\sigma} = \{F_{\mu[\nu}, F_{\rho\sigma]}\}, \tag{2.2c}$$

with anti-hermitean SU(2) gauge potentials A_μ and a doublet Higgs field φ in the fundamental representation of SU(2).

Assuming the necessary asymptotic and smoothness conditions[2)] the model is seen to be topologically nontrivial. $\pi_3(S^3) = \mathbb{Z}$ implies first, that there are pairs of smooth Higgs doublets at infinity which cannot be continuously deformed into each other, and second, that the same is true for group elements Ω which characterize asymptotic pure gauge potentials. However, without the fourth-order terms there cannot be any smooth nontrivial finite-action YMH solutions as already the following simple scaling argument shows: If we substitute:

$$\varphi(x) \longrightarrow \varphi(\lambda x), \quad A_\mu(x) \longrightarrow \lambda A_\mu(\lambda x), \qquad (2.3)$$

we can lower the contribution to the action of the first three YMH terms by a suitable choice of λ. The effect under this rescaling of the additional fourth-order term is to compensate and hence to stabilize the configuration at a finite scale. For this reason we expect to find smooth finite-action solutions.

To construct such a solution we choose the following ansatz:

$$\varphi = h(r)\,\Omega\,\phi_0, \quad A_\mu = [1-k(r)]\,\partial_\mu\Omega\,\Omega^+, \qquad (2.4)$$

$$r^2 = x_\mu x_\mu, \quad \Omega = \hat{x}_\mu q_\mu \in SU(2),$$

$$q = (1_2, i\vec{\sigma}), \quad \phi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

For this field configuration the action reduces to the 1-dimensional integral

$$A = \int_0^\infty dr\, r^3 \Big\{ \frac{12}{r^2}\Big[k'^2 + \frac{4}{r^2} k^2(k-1)^2\Big] + h'^2 + \frac{3}{r^2} h^2 k^2 + (h^2-1)^2 + \frac{6912}{r^6} k^2(k-1)^2 k'^2 \Big\}. \qquad (2.5)$$

Notice that for $k \xrightarrow[r\to\infty]{} 0$, A_μ is pure gauge at infinity, and that the winding number for Ω is one. Given the necessary smoothness conditions, the Pontryagin index is therefore also one.

The variation equations for (2.5) read

$$\frac{1}{r^3}(r^3 h')' = \frac{3}{r^2} h k^2 + 2(h^2-1)\,h, \qquad (2.6a)$$

$$\frac{2304}{r^6}\, k(k-1)\left[k''k(k-1) - \frac{3}{r}k'k(k-1) + k'^2(2k-1)\right]$$
$$+ \frac{4}{r^3}(r k')' = \frac{1}{r^2} h^2 k + \frac{16}{r^4} k(k-1)(2k-1). \tag{2.6b}$$

The important feature of the ansatz (2.4) is that any solution to the equations (2.6) solves the Euler-Lagrange equations for the Lagrangian (2.1). The latter are just the familiar YMH equations augmented by a term resulting from the A_μ variation of the fourth-order term:

$$D_\mu D_\mu \varphi = 2\varphi\,(\langle\varphi,\varphi\rangle - 1),$$
$$D_\nu F_{\nu\mu} + \{D_\nu F_{\nu\mu\rho\sigma}, F_{\rho\sigma}\} = \frac{1}{4}\,\mathrm{Re}\,\langle\varphi, \sigma_a D_\mu \varphi\rangle\, \sigma_a. \tag{2.7}$$

Since every term in (2.5) is positive definite the boundary conditions for finite-action fields are

$$h^2 \xrightarrow[r\to\infty]{} 1; \quad 0,1 \xleftarrow[r\to 0]{} k \xrightarrow[r\to\infty]{} 0. \tag{2.8}$$

The behaviour of k at the origin shows that there are two topologically inequivalent classes. If the minimum in the

topologically nontrivial sector ($k \xrightarrow[r\to 0]{} 1$) is attained, the minimum configuration is a solution. Because of the above scaling argument there is no reason to doubt that a nontrivial configuration attains the minimum. To give a mathematically rigorous proof of this statement one would have to adapt the technique of Tyupkin, Fateev and Shvarts[8] to our case.

Finally, we extract the asymptotic form of the solution to (2.6) at the origin: Because $k \xrightarrow[r\to 0]{} 1$, the finite-action condition guarantees that at worst h goes like $r^{-1+\varepsilon}$ at the origin. Hence, the asymptotic solution to (2.6a) must satisfy

$$r^2 h'' + 3 r h' = 3h, \tag{2.9}$$

which yields $h \sim r$ and guarantees the regularity of the Higgs field (2.4). To check the regularity of the gauge potentials (2.4) we need only consider $k-1 \sim r^\alpha$ for $0 < \alpha < 1$. For this choice of α, only the higher-order terms in (2.6b) contribute at the origin which leads to the equation

$$2\alpha(\alpha - 2) = 0. \tag{2.10}$$

This equation does not have a solution for $0 < \alpha < 1$. Therefore, $\alpha \geq 1$ holds and A_μ is regular.

3. The properties of the ansatz

To show that the features of the mini-model are not accidental and apply to a wide class of models we now discuss in detail the ansatz (2.4) which we rewrite in the form

$$\varphi = h\Omega\phi_0 , \quad A_\mu = \frac{1}{r}(1-k)\,\eta^+_{\mu\nu}\hat{x}_\nu . \tag{3.1}$$

Note that the A_μ of the Belavin-Polyakov-Schwartz-Tyupkin instanton[9] is of the form (3.1). Here we have introduced the antisymmetric (anti-) self-dual tensors

$$\eta^+_{\mu\nu} = q_\mu q^+_\nu - \delta_{\mu\nu} 1_2 , \quad \eta^-_{\mu\nu} = q^+_\mu q_\nu - \delta_{\mu\nu} 1_2 , \tag{3.2}$$

The most important feature of this ansatz is that any gauge invariant action density depends on r only. Equally important, we can show that for a wide class of models the ansatz is compatible, i.e., the equations resulting from variations orthogonal to the ansatz are automatically satisfield.

The properties stated above do not follow directly from the principle of symmetric criticality[10]. This is because the field configuration given by our ansatz is not SO(4) symmetric in the sense that any SO(4) transformation $X_\mu \rightarrow M_{\mu\nu} X_\nu$, $M \in SO(4)$, can be compensated by a SU(2) gauge transformation. In fact, we can easily calculate the compensating SU(2) transformation for φ and then check whether it compensates the same SO(4) transformation on A_μ: For φ the compensating

transformation G must satisfy

$$G M_{\mu\nu}\hat{x}_\nu q_\mu \phi_0 = \hat{x}_\mu q_\mu \phi_0 , \tag{3.3}$$

which yields

$$G = \hat{x}_\mu M_{\mu\nu}\hat{x}_\nu + M_{\mu\nu}\hat{x}_\nu \hat{x}_\rho \eta^+_{\rho\mu} . \tag{3.4}$$

Since G depends on $\hat{x}_\mu$, $\partial_\mu G G^+$ contributes to the transformed A_μ, and G cannot compensate the SO(4) transformation M of A_μ for arbitrary k.

To prove that nevertheless in our case all terms in an arbitrary Lagrange density are $\hat{x}_\mu$ - independent, we define

$$\phi_\perp = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.5}$$

and the real matrices

$$\mathcal{U} = i \langle \phi_0, \eta^- \phi_0 \rangle = -i \langle \phi_\perp, \eta^- \phi_\perp \rangle,$$

$$V = \frac{1}{2}\left(\langle \phi_0, \eta^- \phi_\perp \rangle - \langle \phi_\perp, \eta^- \phi_0 \rangle\right), \qquad (3.6)$$

$$W = \frac{i}{2}\left(\langle \phi_0, \eta^- \phi_\perp \rangle + \langle \phi_\perp, \eta^- \phi_0 \rangle\right).$$

These matrices are antisymmetric and satisfy the following multiplication table:

	U	V	W
U	-1_4	W	-V
V	-W	-1_4	U
W	V	-U	-1_4

Because of the antisymmetry and this multiplication table, the vectors $\hat{x}$, $\hat{u} = U\hat{x}$, $\hat{v} = V\hat{x}$, and $\hat{w} = W\hat{x}$ form an orthonormal basis of R^4. In terms of these matrices and vectors we can now examine every term in any arbitrary gauge invariant action density.

Any such term is a product of terms which are either of the form

$$\operatorname{tr} T_{\mu\nu\ldots} = \langle \phi_0, T_{\mu\nu\ldots} \phi_0 \rangle + \langle \phi_\perp, T_{\mu\nu\ldots} \phi_\perp \rangle, \qquad (3.8a)$$

or of the form

$$\langle D_\mu^{(\lambda)} \varphi, T_{\rho\sigma\ldots} D_\nu^{(\varkappa)} \varphi \rangle, \qquad (3.8b)$$

where T is a product of the gauge fields F and their covariant derivatives and $D_\mu^{(\lambda)}$ stands for a product of covariant derivatives. This means that for the ansatz (3.1) any term in an action density is a sum of terms $f(r) \langle \phi, T_{\mu\nu} \ldots \phi \rangle$ with $\phi = \phi_0$ or $\phi = \phi_\perp$ and with a product $T_{\mu\nu}$ ... of $\hat{x}_\mu$'s and $\eta^-_{\varrho\sigma}$'s. If we now insert the identity $|\phi_0\rangle\langle\phi_0| + |\phi_\perp\rangle\langle\phi_\perp|$ between each pair of η's, each term becomes a product of $\hat{x}_\mu$'s and of matrix elements of U,V and W with r-dependent coefficient functions. Finally we must contract all indices. Using the antisymmetry of the matrices and the multiplication table (3.7) to perform all products of the matrices and of the $\hat{x}_\mu$'s, we can eliminate all $\hat{x}_\mu$ - dependence and are left with functions of r alone.

We have shown that for the ansatz (3.1) the action density is a function of r, h and k alone. Thus, we are left with ordinary differential equations from the variation with respect to h and k. To show that the ansatz is consistent we must show that the ansatz is an extremum with respect to all variations orthogonal to δh and δk as well. Because in our case we cannot apply the principle of symmetric criticality[10] we must discuss the full Euler-Lagrange equations and show that they reduce to differential equations for h and k. So far, we cannot show this in general but only for a special class of models.

First, we show that each term in the variation equation for φ is of the form $f(r)\,\Omega\,\phi_o$. In fact, for each term the scalar product with $<\phi_o, \Omega^+$ is one of the terms discussed above and therefore a function of r alone. On the other hand, the scalar product with $<\phi_\perp, \Omega^+$ contains an odd number of V's or W's which, according to the multiplication table (3.7), we cannot get rid of. Therefore, eventually the antisymmetry of V and W makes this scalar product vanish.

Secondly, we discuss the variation equation for A_μ: All terms which do not contain φ are products of $\hat{x}_\mu$'s and $\bar{\eta}_{\rho\sigma}$'s only with one uncontracted index and r-dependent coefficient function. For these terms we use

$$\frac{1}{2}[\eta^{\pm}_{\mu\nu}, \eta^{\pm}_{\rho\sigma}] = \delta_{\mu\rho}\eta^{\pm}_{\sigma\nu} + \delta_{\mu\sigma}\eta^{\pm}_{\nu\rho} + \delta_{\nu\rho}\eta^{\pm}_{\mu\sigma} + \delta_{\nu\sigma}\eta^{\pm}_{\rho\mu}, \tag{3.9}$$

and

$$\eta^{\pm}_{\mu\rho}\eta^{\pm}_{\rho\nu} = 3\delta_{\mu\nu} + 2\eta^{\pm}_{\mu\nu}, \tag{3.10}$$

to reduce the number of η's to one. Because all terms are su(2) elements, the terms in question must be of the form $f(r)\,\eta^-_{\mu\nu}\hat{x}_\nu$.

We now discuss the terms (3.8b) restricting our attention to action densities in which the terms (3.8b) are only linear. Linear terms lead to terms of the form $\mathrm{Re}<\phi_o, T_{\mu\rho\sigma}\phi_o>\eta^-_{\rho\sigma}$ in the equation of motion. Using (3.9) and (3.10) again, we can reduce the number of η's in $T_{\mu\rho\sigma}$ to at most three. If there is no η in T, the term is of the form $f(r)\,\eta^-_{\mu\nu}\hat{x}_\nu$. If there is only one η, the real part vanishes. For two η's, we use

$$\begin{aligned} &\mathrm{Re}<\phi_o, \eta^{\pm}_{\mu\nu}\eta^{\pm}_{\rho\sigma}\phi_o> \\ &= \frac{1}{2}<\phi_o, \{\eta^{\pm}_{\mu\nu}, \eta^{\pm}_{\rho\sigma}\}\phi_o>, \end{aligned} \tag{3.11}$$

$$\begin{aligned} &\frac{1}{2}\{\eta^{\pm}_{\mu\nu}, \eta^{\pm}_{\rho\sigma}\} \\ &= \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\sigma} \pm \varepsilon_{\mu\nu\rho\sigma}, \end{aligned} \tag{3.12}$$

and the self-duality of η^-. The only remaining term is

$$\begin{aligned} &\hat{x}_\alpha\hat{x}_\beta\hat{x}_\gamma\,\mathrm{Re}<\phi_o, \eta^-_{\mu\alpha}\eta^-_{\rho\beta}\eta^-_{\sigma\gamma}\phi_o>\eta^-_{\rho\sigma} \\ &= \frac{1}{2}\hat{x}_\alpha\hat{x}_\beta\hat{x}_\gamma\,\mathrm{Re}<\phi_o, \eta^-_{\mu\alpha}[\eta^-_{\rho\beta}, \eta^-_{\sigma\gamma}]\phi_o>\eta^-_{\rho\sigma}, \end{aligned}$$

which can be dealt with using eq. (3.9).
The result is again a term of the form $f(r)\,\bar{\eta}_{\mu\nu}\hat{x}_\nu$.

Because of technical difficulties with the above (not very elegant) method we did not check compatibility for other models including the one discussed below. However, we consider the class of models covered in this section wide enough to make (3.1) a very important ansatz. On the other hand, we consider the dimensionally reduced 4-form gauge theory important enough to see how far our technique carries before checking compatibility.

4. The dimensionally reduced 4-form gauge theory

As motivated in Section 1 we are really interested in the dimensionally reduced versions of the higher-order gauge field systems in 4p dimensions, which also have nontrivial topology. Here, we consider the model derived from the 8-dimensional system on $\mathbb{R}^4 \times S^2 \times S^2$ in Ref. 7:

$$
\begin{aligned}
\mathcal{L} = & -\left(\frac{\eta_1^4}{8}+\frac{2\eta_2^4}{g^6}\right)\operatorname{tr} F_{\mu\nu}^2 + \frac{\eta_1^4}{4}\left[\langle D_\mu\varphi, D_\mu\varphi\rangle + \left(\tfrac{1}{2}\eta_2^2 - \langle\varphi,\varphi\rangle\right)\right] \\
& - \langle D_\rho\varphi, F_{\mu\nu}F_{[\mu\nu}D_{\rho]}\varphi\rangle + \frac{1}{24}\operatorname{tr} F_{\mu\nu\rho\sigma}^2 \\
& + \langle\varphi, F_{\mu\nu}^2\varphi\rangle\left(\tfrac{4}{3}\eta_2^2 - \langle\varphi,\varphi\rangle\right) \\
& + \tfrac{4}{3}\eta_2^2\langle D_\mu\varphi, F_{\mu\nu}D_\nu\varphi\rangle - 2\langle\varphi, F_{\mu\nu}\varphi\rangle^2 \\
& - 2\langle D_\mu\varphi, D_\nu\varphi\rangle^2 + \langle D_\mu\varphi, D_\mu\varphi\rangle^2 + \langle D_\mu\varphi, D_\nu\varphi\rangle\langle D_\nu\varphi, D_\mu\varphi\rangle \\
& + 4\,\mathrm{Re}\left[\langle D_\nu\varphi, F_{\mu\nu}\varphi\rangle\langle\varphi, D_\mu\varphi\rangle\right].
\end{aligned}
\tag{4.1}
$$

Note that for simplicity we have put the U(1) field $f_{\mu\nu}$ from Ref. 7 equal to zero.

To reduce the action for our ansatz (3.1) we calculated

$$\langle\varphi,\varphi\rangle = h^2, \quad \langle D_\mu\varphi, D_\mu\varphi\rangle = h'^2 + \frac{3}{r^2} h^2 k^2, \tag{4.2a}$$

$$-\mathrm{tr}\, F_{\mu\nu}^2 = \frac{12}{r^2}\left[k'^2 + \frac{4}{r^2} k^2 (k-1)^2\right], \tag{4.2b}$$

$$-\langle D_\rho\varphi, F_{\mu\nu} F_{[\mu\nu} D_{\rho]}\varphi\rangle$$
$$= \frac{24}{r^4}\left[k' h k + h' k (k-1)\right]^2 + \frac{72}{r^6} h^2 k^4 (k-1)^2,$$

$$\mathrm{tr}\, F_{\mu\nu\rho\sigma}^2 = \frac{6912}{r^6} k'^2 k^2 (k-1)^2,$$

$$-\langle\varphi, F_{\mu\nu}^2\varphi\rangle = \frac{6}{r^2} h^2\left[k'^2 + \frac{4}{r^2} k^2 (k-1)^2\right],$$

$$-\langle\varphi, F_{\mu\nu}\varphi\rangle^2 = \frac{2}{r^2} h^4\left[k'^2 + \frac{4}{r^2} k^2 (k-1)^2\right],$$

$$\langle D_\mu\varphi, D_\nu\varphi\rangle^2 = \left(h'^2 + \frac{3}{r^2} h^2 k^2\right)^2,$$

$$\langle D_\mu\varphi, D_\nu\varphi\rangle\langle D_\nu\varphi, D_\mu\varphi\rangle = \left(h'^2 + \frac{1}{r^2} h^2 k^2\right)^2 + \frac{4}{r^4} h^4 k^4,$$

$$\langle D_\mu\varphi, F_{\mu\nu} D_\nu\varphi\rangle = \frac{6}{r^2} h k\left[h' k' + \frac{2}{r^2} h k^2 (k-1)\right],$$

$$\langle D_\nu\varphi, F_{\mu\nu}\varphi\rangle\langle\varphi, D_\mu\varphi\rangle = -\frac{4}{r^2} h' k' h^3 k + \frac{4}{r^4} h^4 k^3 (k-1).$$

Although it is not obvious from (4.1) and (4.2) we know that the action density is a sum of squares because it stems from $\mathcal{L} = \mathrm{tr}\, F^2_{abcd}$ in 8 dimensions. In particular, we know that the sum of terms in the second to sixth line is positive definite. The submodel we obtain by substituting (4.2) into (4.1) is therefore topologically nontrivial. In fact, (4.2a) and (4.2b) shows that

$$h^2 \xrightarrow[r\to\infty]{} \eta_2^2/2\,; \qquad 0,1 \xleftarrow[r\to 0]{} k \xrightarrow[r\to\infty]{} 0 \tag{4.3}$$

holds. Again the minimum in the topologically nontrivial sector is a solution which can be stabilized by the higher-order terms.

What is left to do is to check compatibility and to give a mathematically rigorous proof for the submodel (4.1) and (4.2) of the Tyupkin-Fateev-Schvarts type[8]. This proof should establish the smoothness of the solution which guarantees that the Pontryagin index is equal to the winding number of Ω and therefore equal to one in our case. Knowing the exact asymptotic behaviour would also make it possible to decide whether the Pontryagin index is equal to the dimensionally reduced fourth Chern-Pontryagin charge from Ref. 7.

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